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THE APPLICATION OF THE CHOQUET INTEGRAL EXPECTED UTILITY IN INTERNATIONAL TRADE

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Abstract. In this paper, we consider the discrete Choquet integral with respect to a fuzzy measure and define the Choquet expected utility as representing an act that utilizes for HS product codes to demonstrate the level of animal product exports between Korea and selected trading partners for years 2010-2013. We also consider the discrete interval-valued Choquet integral with respect to a fuzzy measure and define the interval-valued Choquet expected utility as representing an act that assesses for animal product exports between Korea and trading partners for HS Product Codes $i = 1, 2, 3, 4, 5$.

In particular, we investigate the following applications: (1) the ranking and the level of contribution , from an economic value perspective, for animal exports with HS product code $i = 1, 2, 3, 4, 5$ between Korea and selected trade partners for years 2010-2013 and (2) the ranking and the level of contribution from an economic value perspective for total animal product exports between Korea and its trade partners for both years 2010-2013 and the respective HS product codes $i = 1, 2, 3, 4, 5$.

1. Introduction

The theory of fuzzy sets and interval-valued fuzzy sets has been successfully performed in a myriad of multi-criteria decision making studies (see [3, 5-10, 13, 16, 19]). By using two types of fuzzy sets and Choquet integrals, many researchers have studied the concept of Choquet expected utility and its related areas (see [1, 11, 12, 14-17, 19-21]). Aumann [1], Jang [5-9], Wu-Chen-Nie-Zhang [18], and Zhang-Guo-Liu [21] have studied various integrals of interval-valued functions, for example, the Aumann integrals, the interval-valued fuzzy integral and the interval-valued Choquet integrals. Recently, Wood-Jang [14,15] studied the Choquet integral with respect to an imprecise set function, an imprecise market premium functional which was an interval-valued measure of risk and the Choquet integral with respect to a fuzzy measure of a utility function as aggregation functionals.

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In this paper, we consider the discrete Choquet integral with respect to a fuzzy measure and define the Choquet expected utility $CEU(u(a))$ of a utility u from an act a on S for specified HS product codes for animal product exports between Korea and selected trading partners for years 2010-2013. We also consider the discrete interval-valued Choquet integral with respect to a fuzzy measure and define the interval-valued Choquet expected utility $ICEU(\bar{u}(\bar{a}))$ of a utility $\bar{u}$ from an act $\bar{a}$ on S for animal product exports between Korea and selected trading partners for HS Product Codes $i = 1, 2, 3, 4, 5$.

In particular, we investigate the following applications: (1) the ranking and the level of contribution ,from an economic value perspective, for animal exports with HS product code $i = 1, 2, 3, 4, 5$ between Korea and selected trading partners for years 2010-2013 and (2) the ranking and the level of contribution, from an economic perspective, for total animal product exports between Korea and selected trading partners for both years 2010-2013 and the respective HS product codes $i = 1, 2, 3, 4, 5$.(see [22]).

2. The Choquet expected utility

Let S be a finite set of states of nature and $F(S)$ stands for the set of all fuzzy sets $A = \{(s, f_A(s)) \mid s \in S,\ f_A \longrightarrow [0, 1]\}$. Recall that f_A is called the degree of membership function of A. We first consider a fuzzy measure on S and the Choquet integral with respect to a fuzzy measure of the degree of membership function of f_A as follows.

Definition 2.1. ([3,5-9,13, 15-18, 20, 21]) A fuzzy measure on S is a real-valued function μ on the subsets of S which satisfies

$$\begin{aligned} &\text{(i)} \quad A \subset B \Rightarrow \mu(A) \leq \mu(B); \\ &\text{(ii)} \quad \mu(\emptyset) = 0,\ \mu(S) = 1. \end{aligned} \tag{1}$$

Definition 2.2. ([3,5-9,13, 16-18, 20, 21]) (1) Let $A \in F(S)$. The Choquet integrals with respect to a fuzzy measure μ of a fuzzy set $A = (S, f_A)$ is defined by

$$(C)\int f_A d\mu = \int_0^1 \mu(\{s \in S | f_A(s) \geq \alpha\}) d\alpha, \tag{2}$$

where the integral on the right-hand side is an ordinary one.

(2) Let $S = \{s_1, s_2, \cdots, s_n\}$ be a finite set. The discrete Choquet integral with respect to a fuzzy measure μ is defined by

$$(C)\int f(s) d\mu(s) = \sum_{i=1}^{n} f_A(s^{(i)}) \left[\mu(E^{(i)}) - \mu(E^{(i+1)})\right], \tag{3}$$

where $E^{(i)} = \{s \in S | f_A(s) \geq f_A(s^i)\}$ for $i = 1, 2, \cdots, n$. By convention, let $E^{n+1} = \emptyset$.

We note that the weighted arithmetic mean(WAM) is defined as follows:

$$WAM_w(z_1, \cdots, z_n) = \sum_{i=1}^{n} w_i z_i, \tag{4}$$

with $\sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0$, for $i = 0, 1, \cdots, n$, and the ordered weighted averaging operator(OWA), proposed in 1988 Yager [10] as follows:

$$OWA_w(z_1, \cdots, z_n) = \sum_{i=1}^{n} w_i z^{(i)}, \tag{5}$$

with $\sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0$, for $i = 0, 1, \cdots, n$, and $z^{(1)} \leq z^{(2)} \leq \cdots \leq z^{(n)}$.

Now, we consider the Choquet expected utility(CEU) of a utility u from an act a as follows.

Definition 2.3. Let $u : X \longrightarrow [0, 1]$ be a utility and a be an act from S to X. The Choquet expected utility(CEU) with respect to a fuzzy measure μ of a utility u from an act a is defined by

$$CEU(u(a)) = (C) \int u(a(s)) d\mu(s). \tag{6}$$

We note that if $f_A(s) = u(a(s))$ and $A = (S, f_A)$, then $A \in F(S)$, that is, A is a fuzzy set. Since S is a finite set, then we can order the range of utility from a given act $a : S \to [0, \infty)$ as follows:

$$u(a(s^{(1)})) \leq u(a(s^{(2)})) \leq \cdots \leq u(a(s^{(n)})), \tag{7}$$

where $u(a(s_1)), u(a(s_2)), \ldots, u(a(s_n))$ is the range of utility yielded by act a. From Definition 2.3 and (5), we obtain the following property.

Theorem 2.1. *Let S be a finite of states of nature and X be a finite set of trade values. (1) If $CEU(u(a))$ is the Choquet expected utility, then we have*

$$CEU(u(a)) = \sum_{i=1}^{n} u(a(s^{(i)})) \left[\mu(E^{(i)}) - \mu(E^{(i+1)})\right]. \tag{8}$$

where $E^{(i)} = \{s \in S | u(a(s)) \geq u(a(s^{(i)}))\}$ for all $i = 1, 2, \cdots, n$.

(2) If $u : X \longrightarrow [0, 1]$ is an utility and a is an act from S to X, then there exist a weighted function w and a constant k such that

$$CEU(u(a)) = OWA_w(ku(a)) \tag{9}$$

for all $a \in X$.

Proof. By (1) From (7) and Definition 2.2(2), we get

$$\begin{aligned} CEU(u(a)) &= (C) \int u(a(s)) d\mu(s) \\ &= \sum_{i=1}^{n} u(a(s^{(i)})) \left[\mu(E^{(i)}) - \mu(E^{(i+1)})\right]. \end{aligned} \tag{10}$$

From (7), we see that there exists a constant $k = \sum_{i=1}^{n} \left[\mu_{u(a)}(E^{(i)}) - \mu_{u(a)}(E^{(i+1)})\right]$. If we apply a weight function $w = (w_1, \cdots, w_n)$ is as follows:

$$w_i = \frac{1}{k} \left[\mu_{u(a)}(E^{(i)}) - \mu_{u(a)}(E^{(i+1)})\right], \tag{11}$$

for $i = 1, 2, \cdots, n$, then we see that $k > 0$, $w_i \geq 0$ for all $i = 1, 2, \cdots, n$ and $\sum_{i=1}^{n} w_i = 1$. Thus, by (8), we get

$$\begin{aligned} CEU(u(a)) &= \sum_{i=1}^{n} u(a(s^{(i)})) \left[\mu_{u(a)}(E^{(i)}) - \mu_{u(a)}(E^{(i+1)})\right] \\ &= \sum_{i=1}^{n} u(a(s^{(i)}))kw_i \\ &= \sum_{i=1}^{n} (ku(a))(s^{(i)}) = OWA_w(ku(a)). \end{aligned} \quad (12)$$

From Theorem 2.1 (2), we remark that the Choquet expected utility is a generalization of the OWA operator.

3. The interval-valued Choquet expected utility

Firstly, we consider the operations such as the arithmetic, maximum, minimum, and complement on the set of interval-numbers $IN([0,\infty))$, that is,

$$IN([0,\infty)) = \{\bar{a} = [a_l, a_r] \mid a_l, a_r \in IN([0,\infty)) \text{ and } a_l \leq a_r\}. \quad (13)$$

Definition 3.1. ([5-9]) If $\bar{a} = [a_l, a_r], \bar{b} = [b_l, b_r] \in IN([0,\infty))$ and $k \in [0,\infty)$, then we define arithmetic, maximum, minimum, order, inclusion, and complement operations as follows:
(1) $\bar{a} + \bar{b} = [a_l + b_l, a_r + b_r]$,
(2) $k\bar{a} = [ka_l, ka_r]$,
(3) $\bar{a}\bar{b} = [a_l b_l, a_r b_r]$,
(4) $\bar{a} \vee \bar{b} = [a_l \vee b_l, a_r \vee b_r]$,
(5) $\bar{a} \wedge \bar{b} = [a_l \wedge b_l, a_r \wedge b_r]$,
(6) $\bar{a} \leq \bar{b}$ if and only if $a_l \leq b_l$ and $a_r \leq b_r$,
(7) $\bar{a} < \bar{b}$ if and only if $\bar{a} \leq \bar{b}$ and $\bar{a} \neq \bar{b}$,
(8) $\bar{a} \subset \bar{b}$ if and only if $b_l \leq a_l$ and $a_r \leq b_r$,
(9) $\bar{a}^c = [1 - a_r, 1 - a_l]$ is the complement of $\bar{a}$.

Note that if we take $a \in [0,\infty)$, then $a = [a, a] \in IN[0,\infty)$. We consider an interval-valued fuzzy sets as follows: IF(S) stands for the set of all interval-valued fuzzy sets $\bar{A}$ as follows:

$$\bar{A} = \{(s, f_{\bar{A}}(s)) \mid s \in S \text{ and } f_{\bar{A}} : S \longrightarrow IN([0,1]) \text{ is an interval} - \text{valued function }\}, \quad (14)$$

where $f_{\bar{A}}(s) = [f_{A_l}(s), f_{A_r}(s)]$ is the membership function of an interval-valued fuzzy set. Note that $IN([0,1]) \subset IN([0,\infty))$. Secondly, we define the operations such as arithmetic, maximum, minimum, and complement on $IF(S)$ as follows:

Definition 3.2. If $\bar{A} = [A_l, A_r], \bar{B} = [B_l, B_r] \in IF(S)$, $s \in S$, and $k \in [0,\infty)$, then we define arithmetic, maximum, minimum, order, inclusion, complement operations as follows:
(1) $f_{\bar{A}+\bar{B}}(s) = [f_{A_l+B_l}(s), f_{A_r+B_r}(s)]$,
(2) $f_{k\bar{A}} = kf_{\bar{A}}$,
(3) $f_{\bar{A}}f_{\bar{B}} = [f_{A_l}f_{B_l}, f_{A_r}f_{B_r}]$,
(4) $f_{\bar{A}} \vee f_{\bar{B}} = [f_{A_l} \vee f_{B_l}, f_{A_r} \vee f_{B_r}]$,

(5) $f_{\bar{A}} \wedge f_{\bar{B}} = [f_{A_l} \wedge f_{B_l}, f_{A_r} \wedge f_{B_r}]$,
(6) $f_{\bar{A}} \leq f_{\bar{B}}$ if and only if $f_{A_l} \leq f_{B_l}$ and $f_{A_r} \leq f_{B_r}$,
(7) $f_{\bar{A}} < f_{\bar{B}}$ if and only if $f_{\bar{A}} \leq f_{\bar{B}}$ and $f_{\bar{A}} \neq f_{\bar{B}}$,
(8) $f_{\bar{A}} \subset f_{\bar{B}}$ if and only if $f_{B_l} \leq f_{A_l}$ and $f_{A_r} \leq f_{B_r}$,
(9) $f_{\bar{A}^c} = [1 - f_{A_r}, 1 - f_{A_l}]$ is the complement of $f_{\bar{A}}$.

By using Definition 2.2 (2), Definition 3.1, and Definition 3.2, we define the interval-valued Choquet expected utility(ICEU) of an interval-valued utility U from an interval-valued act $\bar{a} = [a_l, a_r]$ as follows:

Definition 3.3. If $S = \{s_1, s_2, \cdots, s_n\}$ is a finite set, and $\bar{A} = (S, f_{\bar{A}}) \in IF(S)$, then we have

$$(C)\int f_{\bar{A}} d\mu = \sum_{i=1}^{n} f_{\bar{A}}(s^{(i)}) \left(\mu(E_I^{(i)}) - \mu(E_I^{(i+1)})\right), \tag{15}$$

where $E_I^{(i)} = \{s \in S | f_{\bar{A}}(s) \geq f_{\bar{A}}(s^{(i)})\}$ for $i = 1, 2, \cdots, n$. By convention, let $E_I^{n+1} = \emptyset$.

From Definition 3.1 and Definition 3.2 , we observe that

$$f_{\bar{A}}(s^{(1)}) \leq f_{\bar{A}}(s^{(2)}) \leq \cdots \leq f_{\bar{A}}(s^{(n)}) \tag{16}$$

if and only if

$$f_{A_l}(s^{(1)}) \leq f_{A_l}(s^{(2)}) \leq \cdots \leq f_{A_l}(s^{(n)}) \text{ and } f_{A_r}(s^{(1)}) \leq f_{A_r}(s^{(2)}) \leq \cdots \leq f_{A_r}(s^{(n)}) \tag{17}$$

By (16) and (17), we obtain the following theorem.

Theorem 3.1. *If $S = \{s_1, s_2, \cdots, s_n\}$ is a finite set and $\bar{A} = (S, f_{\bar{A}}) \in IF(S)$, then we have*

$$(C)\int f_{\bar{A}} d\mu = \sum_{i=1}^{n} \left[f_{A_l}(s^{(i)}) \left(\mu(E_l^{(i)}) - \mu(E_l^{(i+1)})\right), f_{A_r}(s^{(i)}) \left(\mu(E_r^{(i)}) - \mu(E_r^{(i+1)})\right)\right], \tag{18}$$

where $E_l^{(i)} = \{s \in S | f_{A_l}(s) \geq f_{A_l}(s^{(i)})\}$ and $E_r^{(i)} = \{s \in S | f_{A_r}(s) \geq f_{A_r}(s^{(i)})\}$ for $i = 1, 2, \cdots, n$, $E_l^{n+1} = \emptyset$, and $E_r^{n+1} = \emptyset$. Furthermore, we have

$$(C)\int f_{\bar{A}} d\mu = \left[(C)\int f_{A_l} d\mu, (C)\int f_{A_r} d\mu\right]. \tag{19}$$

Proof. From (16) and (17), we see that $E_I^{(i)} = E_l^{(i)} = E_r^{(i)}$. Thus, we get

$$\begin{aligned}
& (C)\int f_{\bar{A}} d\mu \\
= & \sum_{i=1}^{n} f_{\bar{A}}(s^{(i)}) \left(\mu(E_I^{(i)}) - \mu(E_I^{(i+1)})\right) \\
= & \sum_{i=1}^{n} \left[f_{A_l}(s^{(i)}), f_{A_r}(s^{(i)})\right] \left(\mu(E_I^{(i)}) - \mu(E_I^{(i+1)})\right) \\
= & \sum_{i=1}^{n} \left[f_{A_l}(s^{(i)}) \left(\mu(E_I^{(i)}) - \mu(E_I^{(i+1)})\right), f_{A_r}(s^{(i)}) \left(\mu(E_I^{(i)}) - \mu(E_I^{(i+1)})\right)\right]
\end{aligned}$$

$$= \sum_{i=1}^{n} \left[f_{A_l}(s^{(i)}) \left(\mu(E_l^{(i)}) - \mu(E_l^{(i+1)}) \right), f_{A_r}(s^{(i)}) \left(\mu(E_r^{(i)}) - \mu(E_r^{(i+1)}) \right) \right]. \quad (20)$$

By (20) and Definition 2.2 (2), we have

$$\begin{aligned} & (C) \int f_{\bar{A}} d\mu \\ = & \sum_{i=1}^{n} \left[f_{A_l}(s^{(i)}) \left(\mu(E_l^{(i)}) - \mu(E_l^{(i+1)}) \right), f_{A_r}(s^{(i)}) \left(\mu(E_r^{(i)}) - \mu(E_r^{(i+1)}) \right) \right] \\ = & \left[(C) \int f_{A_l} d\mu, (C) \int f_{A_r} d\mu \right]. \end{aligned} \quad (21)$$

By using Definition 3.3 and Theorem 3.1, we define the interval-valued Choquet expected utility(ICEU) of an interval-valued utility $\bar{u}$ from an interval-valued act $\bar{a} = [a_l, a_r]$ as follows:

Definition 3.4. Let $\bar{a} = [a_l, a_r]$ be an interval-valued act from S to $IN(X) \subset IN([0, \infty))$ and $\bar{u} : X \longrightarrow IN([0, 1])$ be an interval-valued utility such that $U(\bar{a}(s)) = [u(a_l(s)), u(a_r(s))]$ for all $s \in S$. The interval-valued Choquet expected utility(ICEU) with respect to a fuzzy measure μ of an interval-valued utility $\bar{u}$ from an act $\bar{a}$ is defined by

$$ICEU(\bar{u}(\bar{a})) = (C) \int \bar{u}(\bar{a}(s)) d\mu(s). \quad (22)$$

We note that if $f_{\bar{A}}(s) = \bar{u}(\bar{a}(s))$ and $\bar{A} = (S, f_{\bar{A}})$, then $\bar{A} \in IF(S)$, that is, $\bar{A}$ is an interval-valued fuzzy set. Since S is a finite set, then we can order the range of an interval-valued utility $\bar{u}$ from a given interval-valued act $\bar{a} : S \to IN(X) \subset IN([0, \infty))$ as follows:

$$\bar{u}(\bar{a}(s^{(1)})) \leq \bar{u}(\bar{a}(s^{(2)})) \leq \cdots \leq \bar{u}(\bar{a}(s^{(n)})), \quad (23)$$

where $\bar{u}(\bar{a}(s_1)), \bar{u}(\bar{a}(s_2)), \ldots, \bar{u}(\bar{a}(s_n))$ is the range of an interval-valued utility yielded by an interval-valued act $\bar{a}$. From Definition 3.4, Theorem 3.1 and (16), we obtain the following property.

Theorem 3.2. *Let S be a finite of states of nature, $IN(X)$ be a finite set of interval-valued trade values, $\bar{a} = [a_l, a_r]$ be an interval-valued act from S to $IN(X)$, and $\bar{u} = [u_l, u_r] : X \longrightarrow IN([0, 1])$ be an interval-valued utility.*

(1) If $ICEU(\bar{u}(\bar{a}))$ is the interval-valued Choquet expected utility, then we have

$$\begin{aligned} & ICEU(\bar{u}(\bar{a})) \\ = & \sum_{i=1}^{n} \left[u_l(a_l(s^{(i)})) \left(\mu(E_l^{(i)}) - \mu(E_l^{(i+1)}) \right), u_r(a_r(s^{(i)})) \left(\mu(E_r^{(i)}) - \mu(E_r^{(i+1)}) \right) \right], \end{aligned} \quad (24)$$

where $E_l^{(i)} = \{s \in S | u_l(a_l(s)) \geq u_l(a_l(s^{(i)}))\}$ and $E_r^{(i)} = \{s \in S | u_r(a_r(s)) \geq u_r(a_r(s^{(i)}))\}$ for $i = 1, 2, \cdots, n$, $E_l^{n+1} = \emptyset$, and $E_r^{n+1} = \emptyset$.

(2) An interval-valued Choquet expected utility is an ordered weighted averaging operator, that is, there exist an interval-valued weighted function $\bar{w} = [w_l, w_r]$ and an interval number $\bar{k} = [k_l, k_r]$ such that $\sum_{i=0}^{n} \bar{w}_i = 1$ and

$$ICEU(\bar{u}(\bar{a})) = \left[OWA_{w_{l,i}}, OWA_{w_{r,i}} \right] [(k_l u(a_l)), (k_r u(a_r))] \quad (25)$$

for all $\bar{a} \in IN(X)$.

Proof. We note that $\bar{u}(\bar{a}(s^{(i)})) = [u_l(a_l(s)), u_r(a_r(s))]$ for all $s \in S$.
(1) By Theorem 3.1, we get

$$\begin{aligned} & ICEU(\bar{u}(\bar{a})) \\ = & (C)\int \bar{u}(\bar{a}(s))d\mu(s) \\ = & \sum_{i=1}^{n} \bar{u}(\bar{a}(s^{(i)}))\left[\mu_{u(a)}(E_I^{(i)}) - \mu(E_I^{(i+1)})\right] \\ = & \sum_{i=1}^{n}\left[u_l(a_l(s^{(i)}))\left(\mu(E_l^{(i)}) - \mu(E_l^{(i+1)})\right), u_r(a_r(s^{(i)}))\left(\mu(E_r^{(i)}) - \mu(E_r^{(i+1)})\right)\right], \quad (26) \end{aligned}$$

(2) By Theorem 2.1 (2), there exist constants k_l and k_r such that

$$\begin{aligned} & k_l = \sum_{i=1}^{n}\left[\mu(E_l^{(i)}) - \mu(E_l^{(i+1)})\right], \text{ and} \\ & CEU(u_l(a_l)) = OWA_{w_{l,i}}(k_l u(a_l)) \end{aligned} \quad (27)$$

and

$$\begin{aligned} & k_r = \sum_{i=1}^{n}\left[\mu(E_r^{(i)}) - \mu(E_r^{(i+1)})\right], \text{ and} \\ & CEU(u_r(a_r)) = OWA_{w_{r,i}}(k_r u(a_r)), \end{aligned} \quad (28)$$

and $k_l \leq k_r$. If we put an interval number $\bar{k} = [k_l, k_r]$ and an interval-valued weight function $\bar{w} = (\bar{w}_1, \cdots, \bar{w}_n)$ as follows:

$$w_{l,i} = \frac{1}{k_l}\left(\mu(E_l^{(i)}) - \mu(E_l^{(i+1)})\right), \text{ and } w_{r,i} = \frac{1}{k_r}\left(\mu(E_r^{(i)}) - \mu(E_r^{(i+1)})\right) \quad (29)$$

for $i = 1, 2, \cdots, n$, then we see that $k_l, k_r \geq 0$, $w_{l,i} \geq 0$ and $w_{l,i} \geq 0$ for all $i = 1, 2, \cdots, n$ and $\sum_{i=1}^{n} \bar{w}_i = 1$. Thus, by (27) and (28), we get

$$\begin{aligned} ICEU(\bar{u}(\bar{a})) &= [CEU(u_l(a_l)), CEU(u_r(a_r))] \\ &= \left[OWA_{w_{l,i}}(k_l u(a_l)), OWA_{w_{r,i}}(k_r u(a_r))\right] \\ &= \left[OWA_{w_{l,i}}, OWA_{w_{r,i}}\right][(k_l u(a_l)), (k_r u(a_r))]. \end{aligned} \quad (30)$$

We consider an interval-valued ordered averaging operator $IOWA_{\bar{w}}(\bar{u}(\bar{a}))$ defined by

$$IOWA_{\bar{w}}(\bar{u}(\bar{a})) = \left[OWA_{w_{l,i}}, OWA_{w_{r,i}}\right][(k_l u(a_l)), (k_r u(a_r))]. \quad (31)$$

From Theorem 3.2 (2) and (31), we note that the interval-valued Choquet expected utility is a generalization of IOWA operator.

4. Some applications

Two forms of Choquet expected utility are examined in this study. The Choquet expected utility (CEU) is the Choquet integral of a utility on the set of trade values (in USD) that represent the trading relationship that Korea shares with selected trading partners (i.e. Korea-USA, Korea-New Zealand, Korea-India, and Korea-Turkey). In this instance, we examine these respective trading relationships by incorporating a clearly for a defined set of Harmonized System (HS) product code product categories (i.e. HS Codes $i = 1, 2, 3, 4, 5$) for each individual year that is under review (i.e. $2010, 2011, 2012, 2013$). While the interval-valued Choquet expected utility (ICEU) illustrates the total volume of trade that has occurred for a

Harmonized System product code (HS Codes $i = 1, 2, 3, 4, 5$) between Korea and its trading partners (i.e. Korea-USA, Korea-New Zealand, Korea-India, and Korea-Turkey) over a specified time frame (i.e. the aggregate for the $2010-2013$ period) Under the ICEU framework, the comparative assessment doesn't compare the individual value of a Harmonized System product code (i.e. HS Codes $i = 1, 2, 3, 4, 5$) between a country and its various trading partners for a year as it is concerned with the aggregated total for a set of years.

Note that the product code definitions have been provided by the UN Comtrade's online database and the relevant categories are defined as follows:

1. Live animals; animal products.
2. Meat and edible meat offal.
3. Fish and crustaceans, mollusks and other aquatic invertebrates.
4. Dairy produce; birds' eggs; natural honey; edible products of animal origin, not elsewhere specified or included.
5. Products of animal origin, not elsewhere specified or included.

Firstly, we denote that HSPC=HS Product Code, s=Year, $a(s)$=Trade Value, $u(a(s))$=the utility of $a(s)$, $CEU(u,a)$=the Choquet Expected Utility of u from a. By using the trade values in tables A1 A4, we can calculate the Choquet integral of an utility on the set of trade values (in USD) that represent Korea's trading relationship with a particular country for years $2010, 2012, 2012, 2013$. Let $s_1 = 2010, s_2 = 2011, s_3 = 2012, s_4 = 2013$. If we define a fuzzy measure μ_1 on S as follows:

$$\begin{aligned}&\mu_1(E^{(4)}) = \mu_1(\{s^{(4)}\}) = 0.1,\ \ \mu_1(E^{(3)}) = \mu_1(\{s^{(3)}, s^{(4)}\}) = 0.3,\\ &\mu_1(E^{(2)}) = \mu_1(\{s^{(2)}, s^{(3)}, s^{(4)}\}) = 0.6,\ \ \mu_1(E^{(1)}) = \mu_1(\{s^{(4)}, s^{(3)}, s^{(2)}, s^{(1)}\}) = 1,\end{aligned} \tag{32}$$

and if $a(s)$ is the trade value of s and $u(a) = \sqrt{\frac{a}{100141401}}$, then we obtain the following $CEU(u(a))$ as follows:

$$\begin{aligned} CEU(u(a)) &= \sum_{i=1}^{4} u(a(s^{(i)}))\left(\mu_1(E^{(i)}) - (\mu_1(E^{(i+1)})\right) \\ &= 0.4u(a(s^{(1)})) + 0.3u(a(s^{(2)})) + 0.2u(a(s^{(3)})) + 0.1u(a(s^{(4)})). \end{aligned} \tag{33}$$

We note that Equation (33) is different to the arithmetic mean(AM) as follows:

$$AM(u(a)) = 0.25u(a(s^{(1)})) + 0.25u(a(s^{(2)})) + 0.25u(a(s^{(3)})) + 0.25u(a(s^{(4)})). \tag{34}$$

By using equation (33), we obtained Table A1: the Choquet expected utility $CEU(u(a))$ of an utility u from an act a on S for animal product export between Korea and USA for years 2010-2013. Let $CEU_{(i,USA)}(u(a))$ be the Choquet expected utility of a utility u from an act a on S for HS product code $i = 1, 2, 3, 4, 5$ between Korea and the USA.

Thus, we derive the order of the Choquet expected utility $CEU_{(i,USA)}(u(a))$ for HS product code $i = 1, 2, 3, 4, 5$ between Korea and USA for years 2010-2013 as follows:

$$\begin{aligned} CEU_{(2,USA)}(u(a)) &< CEU_{(5,USA)}(u(a)) < CEU_{(1,USA)}(u(a)) \\ &< CEU_{(4,USA)}(u(a)) < CEU_{(3,USA)}(u(a)) \end{aligned} \tag{35}$$

and we derive the ratio of the Choquet expected utility $CEU_{(i,USA)}(u(a))$ for HS product code $i = 1, 2, 3, 4, 5$ between Korea and USA for years 2010-2013 as follows:

$$\begin{aligned} CEU_{(1,USA)}(u(a)) &: CEU_{(2,USA)}(u(a)) : CEU_{(3,USA)}(u(a)) \\ &: CEU_{(4,USA)}(u(a)) : CEU_{(5,USA)}(u(a)) \\ &= 5.66 : 4.48 : 93.88 : 20.82 : 4.85. \end{aligned} \tag{36}$$

Table A1: The CEU for animal product exports between Korea and the USA for years 2010-2013

HSPC	s	$a(s)$(USD)	$u(a(s))$	$CEU_{(i,USA)}(u(a))$
1	s_1	$286892 = a(s^{(1)})$	0.05352	0.05664
	s_2	$330299 = a(s^{(2)})$	0.05743	
	s_3	$358496 = a(s^{(3)})$	0.05983	
	s_4	$364918 = a(s^{(4)})$	0.06037	
2	s_1	$997539 = a(s^{(4)})$	0.09981	0.04483
	s_2	$376805 = a(s^{(3)})$	0.06034	
	s_3	$30005 = a(s^{(1)})$	0.01731	
	s_4	$272884 = a(s^{(2)})$	0.05220	
3	s_1	$74866073 = a(s^{(1)})$	0.86464	0.93879
	s_2	$95654573 = a(s^{(2)})$	0.97734	
	s_3	$100141401 = a(s^{(4)})$	1.00000	
	s_4	$99871717 = a(s^{(3)})$	0.99865	
4	s_1	$3722326 = a(s^{(1)})$	0.19280	0.20821
	s_2	$4323214 = a(s^{(2)})$	0.20778	
	s_3	$5016833 = a(s^{(4)})$	0.22382	
	s_4	$4910771 = a(s^{(3)})$	0.22145	
5	s_1	$235669 = a(s^{(2)})$	0.04851	0.04858
	s_2	$359747 = a(s^{(3)})$	0.05994	
	s_3	$101795 = a(s^{(1)})$	0.05994	
	s_4	$863858 = a(s^{(4)})$	0.09088	

By using equation (33), we obtained Table A2: the Choquet expected utility $CEU(u(a))$ of an utility u from an act a on S for animal product export between Korea and New Zealand(NZ) for years 2010-2013.

Thus, we derive the order of the Choquet expected utility $CEU_{(i,NZ)}(u(a))$ for HS product code $i = 1, 2, 3, 4, 5$ between Korea and New Zealand(NZ) for years 2010-2013 as follows:

$$\begin{aligned} CEU_{(2,NZ)}(u(a)) &< CEU_{(5,NZ)}(u(a)) < CEU_{(1,NZ)}(u(a)) \\ &< CEU_{(4,NZ)}(u(a)) < CEU_{(3,NZ)}(u(a)) \end{aligned} \tag{37}$$

and we derive the ratio of the Choquet expected utility $CEU_{(i,USA)}(u(a))$ for HS product code $i = 1, 2, 3, 4, 5$ between Korea and USA for years 2010-2013 as follows:

$$\begin{aligned} CEU_{(1,NZ)}(u(a)) &: CEU_{(2,NZ)}(u(a)) : CEU_{(3,NZ)}(u(a)) \\ &: CEU_{(4,NZ)}(u(a)) : CEU_{(5,NZ)}(u(a)) \\ &= 0.53 : 0.00 : 78.87 : 15.97 : 1.56. \end{aligned} \tag{38}$$

We remark that in Table A2, $CEU_{(5,NZ)}(u(a))$ is different to the arithmetic mean $AM_{(5,NZ)}(u(a))$ as follows:

$$CEU_{(5,NZ)}(u(a)) = 0.01557 < 0.02728 = AM_{(5,NZ)}(u(a)). \tag{39}$$

By using equation (33), we obtained Table A3: the Choquet expected utility $CEU(u(a))$ of an utility u from an act a on S for animal product exports between Korea and Turkey for years 2010-2013.

Thus, we derive the order of the Choquet expected utility $CEU_{(i,TR)}(u(a))$ for HS product code $i = 1, 2, 3, 4, 5$ between Korea and Turkey(TR) for years 2010-2013 as follows:

$$CEU_{5,TR)}(u(a)) = CEU_{(4,TR)}(u(a)) = CEU_{(2,TR)}(u(a))$$

Table A2: The CEU for animal product exports between Korea and New Zealand for years 2010-2013

HSPC	s	$a(s)$(USD)	$u(a(s))$	$CEU_{(i,NZ)}(u(a))$
1	s_1	$6650 = a(s^{(4)})$	0.00815	0.00533
	s_2	$4497 = a(s^{(3)})$	0.00670	
	s_3	$1589 = a(s^{(1)})$	0.00398	
	s_4	$2779 = a(s^{(2)})$	0.00527	
2	s_1	$0 = a(s^{(1)})$	0.00000	0.00000
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$0 = a(s^{(3)})$	0.00000	
	s_4	$0 = a(s^{(4)})$	0.00000	
3	s_1	$70759196 = a(s^{(2)})$	0.84059	0.78873
	s_2	$91263506 = a(s^{(4)})$	0.95464	
	s_3	$70763937 = a(s^{(3)})$	0.84062	
	s_4	$46632301 = a(s^{(1)})$	0.68240	
4	s_1	$165773 = a(s^{(3)})$	0.04069	0.15976
	s_2	$113751 = a(s^{(1)})$	0.03370	
	s_3	$148756 = a(s^{(2)})$	0.03854	
	s_4	$277350 = a(s^{(4)})$	0.05263	
5	s_1	$0 = a(s^{(1)})$	0.00000	0.01557
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$218022 = a(s^{(3)})$	0.04666	
	s_4	$393025 = a(s^{(4)})$	0.00265	

$$< \; CEU_{(1,TR)}(u(a)) < CEU_{(3,TR)}(u(a)) \tag{40}$$

and we derive the ratio of the Choquet expected utility $CEU_{(i,USA)}(u(a))$ for HS product code $i = 1, 2, 3, 4, 5$ between Korea and the USA for years 2010-2013 as follows:

$$\begin{aligned} CEU_{(1,TR)}(u(a)) &: CEU_{(2,TR)}(u(a)) : CEU_{(3,TR)}(u(a)) \\ &: CEU_{(4,TR)}(u(a)) : CEU_{(5,TR)}(u(a)) \\ &= 0.15 : 0.00 : 4.57 : 0.00 : 0.00. \end{aligned} \tag{41}$$

By using equation (33), we obtained Table A4: the Choquet expected utility $CEU(u(a))$ of an utility u from an act a on S for animal product export between Korea and India for years 2010-2013.

Thus, we derive the order of the Choquet expected utility $CEU_{(i,IND)}(u(a))$ for HS product code $i = 1, 2, 3, 4, 5$ between Korea and India(IND) for years 2010-2013 as follows:

$$\begin{aligned} CEU_{5,IND)}(u(a)) &< CEU_{(1,IND)}(u(a)) < CEU_{(3,IND)}(u(a)) \\ &< CEU_{(4,IND)}(u(a)) < CEU_{(2,IND)}(u(a)) \end{aligned} \tag{42}$$

and we derive the ratio of the Choquet expected utility $CEU_{(i,IND)}(u(a))$ for HS product code $i = 1, 2, 3, 4, 5$ between Korea and India for years 2010-2013 as follows:

$$\begin{aligned} CEU_{(1,IND)}(u(a)) &: CEU_{(2,IND)}(u(a)) : CEU_{(3,IND)}(u(a)) \\ &: CEU_{(4,IND)}(u(a)) : CEU_{(5,IND)}(u(a)) \\ &= 0.26 : 0.88 : 0.36 : 0.47 : 0.00 \end{aligned} \tag{43}$$

Therefore, from $A1 \sim A4$, we illustrate the ranking of trade values for exports between Korea and trading partners.

Table A3: The CEU for animal product exports between Korea and Turkey for years 2010-2013

HSPC	s	$a(s)$(USD)	$u(a(s))$	$CEU(u(a))$
1	s_1	$0 = a(s^{(1)})$	0.00000	0.00154
	s_2	$6900 = a(s^{(4)})$	0.00830	
	s_3	$150 = a(s^{(2)})$	0.00122	
	S_4	$300 = a(s^{(3)})$	0.00173	
2	s_1	$0 = a(s^{(1)})$	0.00000	0.00000
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$0 = a(s^{(3)})$	0.00000	
	s_4	$0 = a(s^{(4)})$	0.00000	
3	s_1	$0 = a(s^{(1)})$	0.00000	0.04570
	s_2	$672952 = a(s^{(3)})$	0.08198	
	s_3	$2532837 = a(s^{(4)})$	0.15904	
	s_4	$199874 = a(s^{(2)})$	0.04468	
4	s_1	$0 = a(s^{(1)})$	0.00000	0.00000
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$0 = a(s^{(3)})$	0.00000	
	s_4	$0 = a(s^{(4)})$	0.00000	
5	s_1	$0 = a(s^{(1)})$	0.00000	0.00000
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$0 = a(s^{(3)})$	0.00000	
	s_4	$0 = a(s^{(4)})$	0.00000	

For HS Product Code 1, we compare Korea and their respective their respective trading partners as follows:

$$CEU_{1,USA)}(u(a)) < CEU_{(1,NZ)}(u(a)) < CEU_{(1,IND)}(u(a)) < CEU_{(1,TR)}(u(a)) \tag{44}$$

and

$$CEU_{(1,USA)}(u(a)) : CEU_{(1,NZ)}(u(a)) : CEU_{(1,IND)}(u(a)) : CEU_{(1,TR)}(u(a)) = 56.64 : 0.53 : 0.15 : 0.26. \tag{45}$$

For HS Product Code 2, we compare Korea and their respective trading partners as follows:

$$CEU_{2,USA)}(u(a)) < CEU_{(2,IND)}(u(a)) < CEU_{(2,NZ)}(u(a)) = CEU_{(2,TR)}(u(a)) \tag{46}$$

and

$$\begin{aligned} &CEU_{(2,USA)}(u(a)) : CEU_{(2,NZ)}(u(a)) : CEU_{(2,IND)}(u(a)) : CEU_{(2,TR)}(u(a)) \\ &= 44.83 : 0.00 : 0.00 : 0.87. \end{aligned} \tag{47}$$

For HS Product Code 3, we compare Korea and their respective trading partners as follows:

$$CEU_{3,USA)}(u(a)) < CEU_{(3,NZ)}(u(a)) < CEU_{(3,TR)}(u(a)) < CEU_{(3,IND)}(u(a)) \tag{48}$$

and

$$\begin{aligned} &CEU_{(3,USA)}(u(a)) : CEU_{(3,NZ)}(u(a)) : CEU_{(3,IND)}(u(a)) : CEU_{(3,TR)}(u(a)) \\ &= 93.88 : 78.87 : 4.57 : 0.37. \end{aligned} \tag{49}$$

For HS Product Code 4, we compare Korea and their respective trading partners as follows:

$$CEU_{4,USA)}(u(a)) < CEU_{(4,NZ)}(u(a)) < CEU_{(4,IND)}(u(a)) < CEU_{(4,TR)}(u(a)) \tag{50}$$

Table A4: the CEU for Animal product expert between Korea and India for years 2010-2013

HSPC	s	$a(s)$(USD)	$u(a(s))$	$CEU(u(a))$
1	s_1	$1050 = a(s^{(3)})$	0.00324	0.00264
	s_2	$1300 = a(s^{(4)})$	0.00360	
	s_3	$450 = a(s^{(1)})$	0.00212	
	s_4	$700 = a(s^{(2)})$	0.00264	
2	s_1	$35432 = a(s^{(3)})$	0.01881	0.00887
	s_2	$50639 = a(s^{(4)})$	0.02249	
	s_3	$2656 = a(s^{(1)})$	0.00515	
	s_4	$8230 = a(s^{(2)})$	0.00907	
3	s_1	$8695 = a(s^{(4)})$	0.009318	0.00368
	s_2	$5247 = a(s^{(3)})$	0.00724	
	s_3	$0 = a(s^{(1)})$	0.00000	
	s_4	$1865 = a(s^{(2)})$	0.00432	
4	s_1	$0 = a(s^{(1)})$	0.00000	0.00470
	s_2	$21614 = a(s^{(3)})$	0.01469	
	s_3	$30938 = a(s^{(4)})$	0.01758	
	s_4	$0 = a(s^{(2)})$	0.00000	
5	s_1	$0 = a(s^{(1)})$	0.00000	0.00000
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$0 = a(s^{(3)})$	0.00000	
	s_4	$0 = a(s^{(4)})$	0.00000	

and

$$\begin{aligned}&CEU_{(4,USA)}(u(a)) : CEU_{(4,NZ)}(u(a)) : CEU_{(4,IND)}(u(a)) : CEU_{(4,TR)}(u(a))\\&= 20.82 : 15.98 : 0.00 : 0.47.\end{aligned} \tag{51}$$

For HS Product Code 5, we compare Korea and their respective trading partners as follows:

$$CEU_{5,USA)}(u(a)) < CEU_{(5,NZ)}(u(a)) < CEU_{(5,TR)}(u(a)) = CEU_{(5,IND)}(u(a)) \tag{52}$$

and

$$\begin{aligned}&CEU_{(3,USA)}(u(a)) : CEU_{(3,NZ)}(u(a)) : CEU_{(3,IND)}(u(a)) : CEU_{(3,TR)}(u(a))\\&= 4.86 : 1.55 : 0.00 : 0.00.\end{aligned} \tag{53}$$

Secondly, we denote that TP=Trade partner, s=HS Product Code, $\bar{a}(s)$=an interval-valued Trade Value, $\bar{u}(\bar{a}(s))$=the interval-valued utility of $\bar{a}(s)$, $ICEU(\bar{u}(\bar{a}))$=the interval-valued Choquet Expected Utility of $\bar{u}$ from $\bar{a}$. By using the trade values in Table A5, we can calculate the interval-valued Choquet integral of an interval-valued utility on the set of interval-valued trade values (in USD) that represent Korea's trading relationship with trading partners for years 2010, 2012, 2012, 2013. Let $s_1 = Code1, s_2 = Code2, s_3 = Code3, s_4 = Code4, s_5 = Code5$. If we define a fuzzy measure μ_2 on S as follows:

$$\begin{aligned}&\mu_2(\{s^{(5)}\}) = 0.1, \mu_2(\{s^{(4)}, s^{(5)}\}) = 0.2, \mu_2(\{s^{(3)}, s^{(4)}, s^{(5)}\}) = 0.4,\\&\mu_2(\{s^{(2)}, s^{(3)}, s^{(4)}, s^{(5)}\}) = 0.7, \mu_2(\{s^{(1)}, s^{(2)}, s^{(3)}, s^{(4)}, s^{(5)}\}) = 1,\end{aligned} \tag{54}$$

and if $\bar{a}(s)$ is the interval-valued trade value of s and $\bar{u}(\bar{a}) = \left[\sqrt{\frac{a_l}{100141401}}, \sqrt{\frac{a_r}{100141401}}\right]$ is the interval-valued utility, then we obtain the following $ICEU(\bar{u}(\bar{a}))$ as follows:

$$ICEU(\bar{u}(\bar{a}))$$

$$= \sum_{i=1}^{n} \left[u_l(a_l(s^{(i)})) \left(\mu(E_l^{(i)}) - \mu(E_l^{(i+1)}) \right), u_r(a_r(s^{(i)})) \left(\mu(E_r^{(i)}) - \mu(E_r^{(i+1)}) \right) \right]$$
$$= \Big[0.3a_l(s^{(1)} + 0.3a_l(s^{(2)}) + 0.2a_l(s^{(3)} + 0.1a_l(s^{(4)} + 0.1a_l(s^{(5)},$$
$$0.3a_r(s^{(1)} + 0.3a_r(s^{(2)}) + 0.2a_r(s^{(3)} + 0.1a_r(s^{(4)} + 0.1a_r(s^{(5)}, \Big]. \tag{55}$$

Table A5: The ICEU for animal product exports between Korea and selected trading partners for HS Product Codes $i = 1, 2, 3, 4, 5$.

TP	s	$\bar{a}(s)$(USD)	$\bar{u}(\bar{a}(s))$	$ICEU(\bar{u}(\bar{a}))$
USA	s_1	$[286892, 364918] = \bar{a}(s^{(1)})$	[0.01731, 0.06037]	[0.14557, 0.19060]
	s_2	$[30005, 997539] = \bar{a}(s^{(3)})$	[0.05352, 0.09981]	
	s_3	$[74866073, 100141401] = \bar{a}(s^{(5)})$	[0.86464, 1.00000]	
	s_4	$[3722326, 5016833] = \bar{a}(s^{(4)})$	[0.19280, 0.22382]	
	s_5	$[1017895, 863858] = \bar{a}(s^{(2)})$	[0.09288, 0.10082]	
NZ	s_1	$[1589, 6650] = \bar{a}(s^{(2)})$	[0.00398, 0.00815]	[0.08084, 0.11470
	s_2	$[0, 0] = \bar{a}(s^{(1)})$	[0.00000, 0.00000]	
	s_3	$[46632301, 91263506] = \bar{a}(s^{(5)})$	[0.68240, 0.95464]	
	s_4	$[113751, 277350] = \bar{a}(s^{(3)})$	[0.03370, 0.05263]	
	s_5	$[218022, 393025] = \bar{a}(s^{(4)})$	[0.04666, 0.06265]	
TR	s_1	$[150, 6900] = \bar{a}(s^{(4)})$	[0.00122, 0.00830]	[0.00490, 0.01673
	s_2	$[0, 0] = \bar{a}(s^{(1)})$	[0.00000, 0.00000]	
	s_3	$[199874, 2532837] = \bar{a}(s^{(5)})$	[0.04468, 0.15904]	
	s_4	$[0, 0] = \bar{a}(s^{(2)})$	[0.00000, 0.00000]	
	s_4	$[0, 0] = \bar{a}(s^{(3)})$	[0.00000, 0.00000]	
IND	s_1	$[450, 1300] = \bar{a}(s^{(2)})$	[0.00212, 0.00360]	0.00348, 0.00695]
	s_2	$[2656, 50630] = \bar{a}(s^{(5)})$	[0.01469, 0.02249]	
	s_3	$[1865, 8695] = \bar{a}(s^{(3)})$	[0.00432, 0.00932]	
	s_4	$[21614, 30938] = \bar{a}(s^{(4)})$	[0.00515, 0.05551]	
	s_5	$[0, 0] = \bar{a}(s^{(2)})$	[0.00000, 0.00000]	

By using equation (55), we obtained Table A5: the interval-valued Choquet expected utility $ICEU(\bar{u}(\bar{a}))$ of a utility $\bar{u}$ from an act $\bar{a}$ on S, for animal product export between Korea and selected trading partners for HS Product Codes $i = 1, 2, 3, 4, 5$.

We remark that in order to calculate the $ICEU$ in Table A5, we changed four interval-valued trading values for the USA and India as follows:

$$[286892, 364918] = \bar{a}(s^{(1)}) \text{ and } [30005, 997539] = \bar{a}(s^{(3)}) \tag{56}$$

are changed by

$$[30005, 364918] = \bar{a}(s^{(1)}) \text{ and } [286892, 997539] = \bar{a}(s^{(3)}) \tag{57}$$

and

$$[2656, 50630] = \bar{a}(s^{(5)}) \text{ and } [21614, 30938] = \bar{a}(s^{(4)}) \tag{58}$$

are changed by

$$[21614, 50630] = \bar{a}(s^{(5)}) \text{ and } [2656, 30938] = \bar{a}(s^{(4)}). \tag{59}$$

Thus, we note that since $[30005, 364918] \leq [286892, 997539]$ and $[21614, 50630] \geq [2656, 30938]$, we are able to calculate the $ICEU$.

Thus, we derive the order of the Choquet expected utility $ICEU_{USA}(\bar{u}(\bar{a}))$, $ICEU_{NZ}(\bar{u}(\bar{a}))$, $ICEU_{TR}(\bar{u}(\bar{a}))$, and $ICEU_{IND}(\bar{u}(\bar{a}))$ between Korea and selected trading partners for years 2010-2013 and for HS product codes $i = 1, 2, 3, 4, 5$ as follows:

$$ICEU_{IND}(\bar{u}(\bar{a})) < ICEU_{TR}(\bar{u}(\bar{a})) < ICEU_{NZ}(\bar{u}(\bar{a})) < ICEU_{USA}(\bar{u}(\bar{a})) \tag{60}$$

and we derive the ratio of the Choquet expected utility $CEU_{(i,IND)}(u(a))$ for HS product code $i = 1, 2, 3, 4, 5$ between Korea and India for years 2010-2013 as follows:

$$\begin{aligned} & ICEU_{IND}(\bar{u}(\bar{a})) : ICEU_{TR}(\bar{u}(\bar{a})) : ICEU_{NZ}(\bar{u}(\bar{a})) : ICEU_{USA}(\bar{u}(\bar{a})) \\ = \; & [14.56, 19.06] : [8.08, 11.47] : [0.49, 1.67] : [0.34, 0.69]. \end{aligned} \tag{61}$$

5. Conclusions

In this paper, by using the discrete Choquet integral with respect to a fuzzy measure of a fuzzy set (see Definition 2.2 (2) and Theorem 2.1), we obtained a new usage for the Choquet expected utility(see Definition 2.3). This CEU is provides a useful means of investigating both the ranking and contribution of animal exports between Korea and selected trading partners (see Tables $A1 \sim A4$).

The inequalities (35), (37), (40), and (42) rank the order of contribution to animal exports values for HS product codes $i = 1, 2, 3, 4, 5$ between Korea and selected trading partners for years 2010-2013.

The proportional expressions (36), (38), (41), and (43) rank the level of contribution to animal exports for HS product codes $i = 1, 2, 3, 4, 5$ between Korea and selected trading partners for years 2010-2013.

For HS product codes $i = 1, 2, 3, 4, 5$, the inequalities (44), (46), (48), (50), and (52) rank the order of contribution to animal export values between Korea and selected trading partners for years 2010-2013.

For HS product codes $i = 1, 2, 3, 4, 5$, the proportional expressions (45), (47), (49), (51), and (53) rank the level of contribution to animal export values between Korea and selected trading partners for years 2010-2013.

We also defined the discrete interval-valued Choquet integral with respect to a fuzzy measure of an interval-valued fuzzy set (see Definition 3.3). By using the discrete interval-valued Choquet integral and Theorem 3.1, we obtained another new usage for the interval-valued Choquet expected utility(see Definition 3.4). This ICEU provides a useful tool from which we can investigate the ranking and the level of contributions for total animal export valued between Korea and selected trading partners for both years 2010-2013 and HS product code $i = 1, 2, 3, 4, 5$ (see Tables $A5$). Inequality (60) rank the order of contribution to total animal export values between Korea and selected trading partners for both years 2010-2013 and HS product codes $i = 1, 2, 3, 4, 5$ The proportional expression (61) rank the level of contribution for total animal export values between Korea and selected trading partners for both years 2010-2013 and HS product code $i = 1, 2, 3, 4, 5$.

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